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Standard Practice for Reporting Uniaxial Strength Data and Estimating Weibull Distribution Parameters for Advanced Graphites[1]

This standard is issued under the fixed designation D7846; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This practice covers the reporting of uniaxial strength data for graphite and the estimation of probability distribution parameters for both censored and uncensored data. The failure strength of graphite materials is treated as a continuous random variable. Typically, a number of test specimens are failed in accordance with the following standards: Test Methods C565, C651, C695, C749, Practice C781 or Guide D7775. The load at which each specimen fails is recorded. The resulting failure stresses are used to obtain parameter estimates associated with the underlying population distribution. This practice is limited to failure strengths that can be characterized by the two-parameter Weibull distribution. Furthermore, this practice is restricted to test specimens (primarily tensile and flexural) that are primarily subjected to uniaxial stress states.

1.2 Measurements of the strength at failure are taken for various reasons: a comparison of the relative quality of two materials, the prediction of the probability of failure for a structure of interest, or to establish limit loads in an application. This practice provides a procedure for estimating the distribution parameters that are needed for estimating load limits for a particular level of probability of failure.

1.3 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

C565 Test Methods for Tension Testing of Carbon and Graphite Mechanical Materials

C651 Test Method for Flexural Strength of Manufactured Carbon and Graphite Articles Using Four-Point Loading at Room Temperature

C695 Test Method for Compressive Strength of Carbon and Graphite

C749 Test Method for Tensile Stress-Strain of Carbon and Graphite

C781 Practice for Testing Graphite Materials for Gas-Cooled Nuclear Reactor Components

D4175 Terminology Relating to Petroleum Products, Liquid Fuels, and Lubricants

D7775 Guide for Measurements on Small Graphite Specimens

E6 Terminology Relating to Methods of Mechanical Testing

E178 Practice for Dealing With Outlying Observations

E456 Terminology Relating to Quality and Statistics

3. Terminology

3.1 Proper use of the following terms and equations will alleviate misunderstanding in the presentation of data and in the calculation of strength distribution parameters.

3.2 *Definitions:*

3.2.1 *estimator, n*—a well-defined function that is dependent on the observations in a sample. The resulting value for a given sample may be an estimate of a distribution parameter (a point estimate) associated with the underlying population. The arithmetic average of a sample is, for example, an estimator of the distribution mean.

3.2.2 *population, n*—the totality of valid observations (performed in a manner that is compliant with the appropriate test standards) about which inferences are made.

3.2.3 *population mean, n*—the average of all potential measurements in a given population weighted by their relative frequencies in the population.

3.2.4 *probability density function, n*—the function $f(x)$ is a probability density function for the continuous random variable X if:

$$f(x) \geq 0 \quad (1)$$

and

[1] This practice is under the jurisdiction of ASTM Committee D02 on Petroleum Products, Liquid Fuels, and Lubricants and is the direct responsibility of Subcommittee D02.F0 on Manufactured Carbon and Graphite Products.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad (2)$$

The probability that the random variable X assumes a value between a and b is given by:

$$\Pr(a < X < b) = \int_{a}^{b} f(x)\,dx \quad (3)$$

3.2.5 *sample, n*—a collection of measurements or observations taken from a specified population.

3.2.6 *skewness, n*—a term relating to the asymmetry of a probability density function. The distribution of failure strength for graphite is not symmetric with respect to the maximum value of the distribution function; one tail is longer than the other.

3.2.7 *statistical bias, n*—inherent to most estimates, this is a type of consistent numerical offset in an estimate relative to the true underlying value. The magnitude of the bias error typically decreases as the sample size increases.

3.2.8 *unbiased estimator, n*—an estimator that has been corrected for statistical bias error.

3.2.9 *Weibull distribution, n*—the continuous random variable X has a two-parameter Weibull distribution if the probability density function is given by:

$$f(x) = \left(\frac{m}{S_c}\right)\left(\frac{x}{\beta}\right)^{m-1} \exp\left[-\left(\frac{x}{\beta}\right)^{m}\right]; \quad x>0 \quad (4)$$

$$f(x) = 0;\ x \leq 0$$

and the cumulative distribution function is given by:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{m}\right]; \quad x>0 \quad (5)$$

or

$$F(x) = 0; \quad x \leq 0$$

where:

m = Weibull modulus (or the shape parameter) ($m > 0$), and
β = scale parameter (> 0).

3.2.9.1 *Discussion*—The random variable representing uniaxial tensile strength of graphite will assume only positive values, and the distribution is asymmetrical about the population mean. These characteristics rule out the use of the normal distribution (as well as others) and favor the use of the Weibull and similar skewed distributions. If the random variable representing uniaxial tensile strength of a graphite is characterized by Eq 4, and Eq 5, then the probability that the tested graphite will fail under an applied uniaxial tensile stress, σ, is given by the cumulative distribution function:

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{S_c}\right)^{m}\right]; \quad \sigma>0 \quad (6)$$

and

$$P_f = 0; \quad \sigma \leq 0$$

where:

P_f = the probability of failure, and
S_c = the Weibull characteristic strength.

3.2.9.2 *Discussion*—The Weibull characteristic strength depends on the uniaxial test specimen (tensile, compression and flexural) and may change with specimen geometry. In addition, the Weibull characteristic strength has units of stress and should be reported using units of MPa or GPa.

3.3 For definitions of other statistical terms, terms related to mechanical testing, and terms related to graphite used in this practice, refer to Terminologies D4175, E6, and E456, or to appropriate textbooks on statistics (**1-5**).[3]

3.4 *Nomenclature:*

$F(x)$ = cumulative distribution function
$f(x)$ = probability density function
$\mathscr{L}$ = likelihood function
m = Weibull modulus
$\hat{m}$ = estimate of the Weibull modulus
$\hat{m}_U$ = unbiased estimate of the Weibull modulus
N = number of specimens in a sample
P_f = probability of failure
t = intermediate quantity used in calculation of confidence bounds
X = random variable
x = realization of a random variable X
β = Weibull scale parameter
$\hat{\mu}$ = estimate of mean strength
σ = uniaxial tensile stress
σ_i = maximum stress in the ith test specimen at failure
S_c = Weibull characteristic strength (associated with a test specimen)
$\hat{S}_c$ = estimate of the Weibull characteristic strength

4. Summary of Practice

4.1 This practice provides a procedure to estimate Weibull distribution parameters from failure data for graphite data tested in accordance with applicable ASTM test standards. The procedure consists of computing estimates of the biased Weibull modulus and Weibull characteristic strength. If necessary, compute an estimate of the mean strength. If the sample of failure strength data is uncensored, compute an unbiased estimate of the Weibull modulus, and compute confidence bounds for both the estimated Weibull modulus and the estimated Weibull characteristic strength. Finally, prepare a graphical representation of the failure data along with a test report.

5. Significance and Use

5.1 Two- and three-parameter formulations exist for the Weibull distribution. This practice is restricted to the two-parameter formulation. An objective of this practice is to obtain point estimates of the unknown Weibull distribution parameters by using well-defined functions that incorporate the failure data. These functions are referred to as estimators. It is desirable that an estimator be consistent and efficient. In addition, the estimator should produce unique, unbiased estimates of the distribution parameters (**6**). Different types of

[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.